

# Data Quality Maximization for Mobile Crowdsensing

Cheng Zhang\*, and Noriaki Kamiyama<sup>†</sup>

\*Department of Computer Science and Communications Engineering, Waseda University, Tokyo, 169-0072 Japan

<sup>†</sup>Department of Information and Communications Engineering, Fukuoka University, Fukuoka, 814-0180 Japan

Email: cheng.zhang@akane.waseda.jp, kamiyama@fukuoka-u.ac.jp

**Abstract**—With the increase of smart devices, mobile crowdsensing, in which a crowdsensing Internet of Things (IoT) platform collects data from smart devices (such as smartphone) users, has become a popular paradigm. Various incentive mechanisms are widely employed for the IoT platform to incentivize smart device users to provide sensing data. Traditional works concentrated on rewarding smart device users for their short term effort to provide data, without considering smart device users' long term factors and the quality of data. In this paper, smart device users' quality of data is considered by incorporating smart device users' long term factor reputation. A quality maximization problem with budget constraints is formulated for IoT platform, and the optimal pricing solution is obtained through theoretical analysis. Our proposed optimal pricing based incentive mechanism is validated by extensive numerical simulations.

**Index Terms**—Mobile crowdsensing, data quality, network economics, convex optimization

## I. INTRODUCTION

Internet of Things (IoT) services are expected to enrich life of people through utilizing various sensing data obtained by many IoT devices [1]. IoT services consist of IoT devices, IoT platforms, and service consumers. IoT devices measure various environmental data, e.g., temperature, humidity, and hazardous substances concentration, and provide the sensed data to the IoT platforms. After analyzing the collected data [2], the IoT platforms extract various information and knowledges which are useful to consumers, and provide various services utilizing the obtained information and knowledges to service consumers.

Sensing functions measuring various environmental data are provided to many smartphones recently. Because of proliferation of smartphones with embedded sensors, mobile users become active contributors providing various kinds of sensing data, and *mobile crowdsensing* has attracted a wide attention as a new scheme, in which smartphones is seen as a kind of IoT devices [3]. However, smartphone users are not under control of the IoT platforms, and they selfishly determine how much effort they make on acquiring the sensing data by smartphones. Sensing data through smartphones requires smartphone users to pay costs of some types, i.e., mobile device energy consumption, mobile operator bill, devotion of precious time, and attention to perform the sensing task. Therefore, to collect sensing data from smartphone users, the

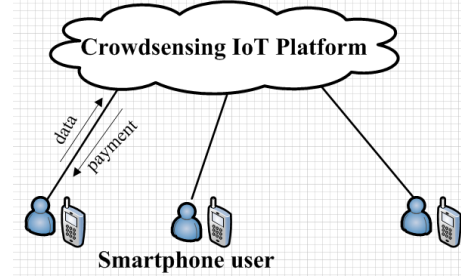


Fig. 1. Smartphone user provides data to IoT platform, and gets rewarded by payment from IoT platform. Smartphone user tries to maximize his/her utility, which is defined as the difference between the payment from IoT platform and cost to generate the data.

IoT platforms are required to pay incentives to smartphone users [4], and many current commercial crowdsensing systems, e.g., Amazon Mechanical Turk, post a fixed price to smartphone users to incentivize participation [5]. However, the quality of sensing data will depend on the effort of smartphone users in crowdsensing, so the effort of smartphone users should be reflected on the incentive paid to smartphone users to give them a motivation to make higher efforts on acquiring sensing data.

By reflecting the quality of sensing data on the price paid to smartphone users, IoT platforms can expect smartphone users to make more effort on improving the quality of sensing data. Therefore, some existing works proposed to give smartphone users incentivizes depending on the data value, i.e., quality of data [6] [7] [8]. The quality of sensing data depends on the ability of smartphone users in sensing data, so the achieved quality is different among smartphone users even when they make the same effort. The IoT platforms can expect to collect sensing data with high quality by smaller amount of payment if they pay the incentive to smartphone users with higher ability to acquire precise data. The total amount of incentives which the IoT platforms can afford paying to smartphone users is limited, so the IoT platforms need to carefully select smartphone users to which incentives are paid to maximize the total quality of sensing data collected under the budget constraint of the IoT platforms. However, existing incentive mechanisms did not consider the ability of smartphone users on sensing data as well as the constraint of the total budget of IoT platform.

In this paper, we propose a new incentive mechanism for smartphone users to provide precise sensing data to the IoT platforms. The contributions of this manuscript are summarized as follows.

- To effectively and efficiently collect sensing data from smartphone users with the constraint of total budget of IoT platforms, we propose to set the payment to smartphone users depending on the ability of acquiring sensing data, or quality of sensing data, which is estimated from the users' reputation and effort.
- We formalize the optimization problem of setting the payment to each user so that the total quality of data obtained by the IoT platforms is maximized under the constraint of the total budget of IoT platforms. We also derive the optimum price by utilizing the Lambert W function and solving the Karush-Kuhn-Tucker (KKT) conditions.
- Through numerical evaluation, we show that the proposed incentive mechanism can improve the total quality of data acquired by IoT platforms.

We organize the rest sections of the paper as the following. Section III is the system model, in which main notation and definition is provided. The quality maximization problem is formulated in Section IV. Then the solution of problem is analyzed in Section V by showing the target problem is a concave optimization problem. Section VI shows the numeric results to valid our theoretic analysis. Finally, section VII concludes this paper and illustrate the future work.

## II. RELATED WORKS

There are some existing works proposed to use prices paid to smartphone users to incentivize them to provide sensing data. Lee et al. proposed a reverse auction model for setting prices [9], and Jin et al. also proposed a double auction model directly matching IoT devices and service consumers [10] [11]. However, the IoT platforms cannot maximize the value of data collected from the smartphone users because smartphone users do not know the quality of data they provide to the IoT platforms. Han et al. considered the effort of smartphone users in sensing data by setting the price posted to smartphone users so that the cost of IoT platforms, i.e., the total amount of incentive paid to smartphone users, was minimized when the distribution of quality of data can be estimated from the efforts of smartphone users [5]. However, the fixed price was posted to all smartphone users, so the IoT platforms cannot incentivize smartphone users to make higher effort to get sensing data.

To incentive smartphone users to make higher effort on acquiring sensing data with higher quality, the price paid to each smartphone user should depend on the quality of sensing data provided to the IoT platforms. Some existing works proposed to set the price based on the value of sensing data. For example, Niyato et al. proposed to set the fee obtained from service consumers based on the value of sensing data which was defined by the detection probability of event

[12]. However, this method cannot be applied for setting the incentive paid to IoT devices, i.e., smartphone users.

Peng et al. proposed to set the incentive based on the estimated quality of sensing data provided by each smartphone user, so smartphone users were motivated to make effort to improve the quality of sensing data [7]. Jin et al. also proposed to set the incentives paid to smartphone users so that the profit of IoT platforms was maximized when that smartphone users maximized their utility by optimizing the variance of error distribution of sensing data which was assume to obey the Gauss distribution [6]. Moreover, Zheng et al. also proposed to set the incentives paid to smartphone users based on the accuracy of sensing data which was estimated by Gaussian Process [8]. However, these methods did not consider both the ability of smartphone users to acquire sensing data and the budget of IoT platforms, so the IoT platforms cannot maximize the total quality of sensing data with the budget constraint.

## III. SYSTEM MODEL

In this paper, we only consider one specific location  $l$ . The smartphone users' set at location  $l$  is denoted as  $\mathcal{N}_l = \{1, \dots, N_l\}$ .  $\mathcal{N}_l^* \subseteq \mathcal{N}_l$  denotes the smartphone users' set in which smartphone users provide data for the IoT platform at location  $l$ . The quality of data is defined as follows in Eq.(1).

$$q_i(s_i) = \gamma_i \ln(1 + s_i) \quad (1)$$

where  $\gamma_i$  is the reputation of smartphone user  $i \in \mathcal{N}_l$ , which is recorded by IoT platform.  $s_i$  is smartphone user  $i$ ' effort to generate data. Please note that we have a potential assumption that the quality of data is depended on user's reputation and effort. The Eq.(1) actually shows the trustiness of a user. The intrinsic quality of data [13] depends not only on the trustiness of a user, but also other factors, such as the technical capabilities of the smartphone, etc. It is left to the future work to consider intrinsic quality of data.

The cost of data is defined as follows in Eq.(2).

$$c_i(s_i) = \lambda_i s_i \quad (2)$$

where  $\lambda_i$  the cost parameter per unit effort.

Smartphone users' utility is defined as follows in Eq.(3).

$$\begin{aligned} u_i(s_i) &= p_i q_i(s_i) - c_i(s_i) \\ &= p_i \gamma_i \ln(1 + s_i) - \lambda_i s_i \end{aligned} \quad (3)$$

where  $p_i$  is the price per unit data quality paid from IoT platform.  $p_i q_i(s_i)$  is the payment by IoT platform, which is proportional to smartphone users' data quality. We will consider other nonlinear pricing scheme in the future work. Please note that IoT platform sets the payment from its budget  $B_l$ . Given the payment from IoT platform, smartphone user  $i$  tries to maximize his/her utility by setting suitable  $s_i$ . Therefore, a utility maximization problem for smartphone users is formulated as follows.

$$\begin{aligned} \max_{s_i} & u_i(s_i) \\ \text{s.t.} & s_i \leq 0 \end{aligned} \quad (4)$$

**Proposition 1:** The solution for problem in Eq.(4) is as follows.

$$s_i^* = \frac{\gamma_i}{\lambda_i} p_i - 1 \quad (5)$$

*Proof:* Since  $\frac{d^2 u_i}{ds_i^2} = -\frac{p_i \gamma_i}{(1+s_i)^2} < 0$ , then optimal solution exists for problem defined in Eq.(4).

By setting  $\frac{du_i}{ds_i} = 0$ , the optimal solution can be obtained as follows.

$$s_i^* = \frac{\gamma_i}{\lambda_i} p_i - 1 \quad (6)$$

#### Q.E.D

#### IV. QUALITY MAXIMIZATION PROBLEM

Given a budget  $B_l$  for location  $l$ , IoT platform tries to maximize the sum of data quality at location  $l$ . A quality maximization problem (QMP) is formulated as follows in Eq.(7).

$$\begin{aligned} \max_{p_i} \quad & \sum_{i \in \mathcal{N}_l^*} q_i(s_i) \\ \text{s.t.} \quad & \sum_{i \in \mathcal{N}_l^*} p_i q_i(s_i) \leq B_l \\ & u_i \geq 0, \forall i \in \mathcal{N}_l \\ & p_i \geq 0, \forall i \in \mathcal{N}_l \end{aligned} \quad (7)$$

substitute  $s_i^* = \frac{\gamma_i}{\lambda_i} p_i - 1$  to the objective function and constraint, we have the following new problem

$$\begin{aligned} \max_{p_i} \quad & \sum_{i \in \mathcal{N}_l^*} q_i(s_i^*) \\ = \max_{p_i} \quad & \sum_{i \in \mathcal{N}_l^*} \gamma_i \ln\left(\frac{\gamma_i}{\lambda_i} p_i\right) \\ \text{s.t.} \quad & \sum_{i \in \mathcal{N}_l^*} p_i q_i(s_i^*) = \sum_{i \in \mathcal{N}_l^*} p_i \gamma_i \ln\left(\frac{\gamma_i}{\lambda_i} p_i\right) \leq B_l \\ & p_i \geq 0, \forall i \in \mathcal{N}_l^* \end{aligned} \quad (8)$$

Please note that the condition  $u_i \geq 0$  in Eq.(7) is no longer needed in the new problem in Eq.(8) since when substitute the optimal  $s_i^*$  to the objective function,  $u_i$  is maximized.

#### V. THEORETIC ANALYSIS

In this section, the problem defined in section IV is theoretically analyzed. Firstly, we proof the objective function of the target problem is concave. Secondly, convex optimization technique is utilized to solve the target optimization problem.

As for the objective function of the problem defined in Eq.(8), we have the following **Proposition 2**.

**Proposition 2:** The objective function of the new problem in Eq.(8) is concave.

Please refer to Appendix VIII-A for the proof of **Proposition 2**.

**Definition 1:** The *Lambert W* function [14] is a set of functions that are the inverse of the function  $f(z) = ze^z$ ,

where  $e^z$  is the exponential function, and  $z$  is any complex number. We can express it as follows.

$$z = f^{-1} = W(ze^z) \quad (9)$$

By substituting  $z_0 = ze^z$  into the Eq.(9), we get the definition of the  $W$  function as follows.

$$z_0 = W(z_0)e^{W(z_0)} \quad (10)$$

Properties of Lambert  $W$  function [14] are as follows.

- Property (1): The exponential of Lambert  $W$  function is as follows.

$$e^{W(z)} = \frac{z}{W(z)} \quad (11)$$

- Property (2): The derivative of Lambert  $W$  function is as follows.

$$\frac{dW}{dz} = \frac{W(z)}{z(1+W(z))} \quad \text{for } z \notin \left\{0, -\frac{1}{e}\right\} \quad (12)$$

**Lemma 1:** As for the following equation in Eq.(13),  $m$  and  $n$  are the constants,  $x$  is the variable.

$$m = \frac{1}{x} \cdot \frac{1}{[\ln(nx) + 1]} \quad (13)$$

Its solution is as follows.

$$x = \frac{e^{W(\frac{ne}{m})}}{ne} \quad (14)$$

or

$$x = \frac{1}{mW(\frac{ne}{m})} \quad (15)$$

where  $W(\cdot)$  is the Lambert  $W$  function defined in **Definition 1**.

Please refer to Appendix VIII-B for the proof of Lemma 1. The solution is illustrated in the following **Proposition 3**.

**Proposition 3:** The solution is for Eq.(7) is  $p_i^* = h_i'(\alpha^*)$ , where function  $h_i'(\alpha^*)$  is defined in Eq.(16), and  $\alpha^*$  is determined by Eq.(17).

$$\begin{aligned} p_i^* &= h_i'(\alpha) \\ &= \frac{\lambda_i e^{W(\frac{e\gamma_i}{\lambda_i\alpha})}}{e\gamma_i} = \frac{1}{\alpha W(\frac{\gamma_i e}{\lambda_i\alpha})} \end{aligned} \quad (16)$$

$$G(\alpha) = B_l \quad (17)$$

where

$$\begin{aligned} G(\alpha) &= \sum_{i \in \mathcal{N}_l^*} \frac{\gamma_i [W(\frac{\gamma_i e}{\lambda_i\alpha}) - 1]}{\alpha [1 + W(\frac{\gamma_i e}{\lambda_i\alpha})]} \\ &+ \sum_{i \in \mathcal{N}_l^*} \left[ \frac{\gamma_i}{\alpha W(\frac{\gamma_i e}{\lambda_i\alpha}) [1 + W(\frac{\gamma_i e}{\lambda_i\alpha})]} \ln\left(\frac{\gamma_i}{\lambda_i} \cdot \frac{1}{\alpha W(\frac{\gamma_i e}{\lambda_i\alpha})}\right) \right] \end{aligned} \quad (18)$$

Please refer to Appendix VIII-C for the proof of **Proposition 3**.



Fig. 2. Budget  $B_l$  and Total data quality vs.  $\alpha$ .

## VI. NUMERIC ANALYSIS

In this section, the performances of our optimal pricing based incentive mechanism are evaluated by comparing them with fixed pricing based incentive mechanism.

We have implemented a simulator by Python 2.7.1. The number of smartphone users  $N$  is assumed as 1000, and the smartphone users' reputation parameter  $\gamma_i$  ( $\forall i \in \mathcal{N}_l$ ) is extracted from uniform distribution  $U[10, 20]$ . smartphone users' cost parameter  $\lambda_i$  ( $\forall i \in \mathcal{N}_l$ ) is extracted from uniform distribution  $U[2, 8]$ .

Firstly, we find the optimal solution by numeric analysis. According to the theoretic analysis in section V, to find the optimal Lagrangian multiplier  $\alpha$  is important to solve the primary problem Eq.(4). It is difficult to get explicit solution for  $\alpha$  by Eq.(43). We set  $\alpha$ 's value from 0.05 to 0.46 by step 0.005, and calculate  $G(\alpha)$ 's value by Eq.(43) (Please note that  $G(\alpha)$ 's value is equal to  $B_l$ ). Fig. 2 shows how the budget  $B_l$  and total data quality changes with different  $\alpha$  from 0.05 to 0.46. Therefore, the optimal solution  $\alpha^*$  can easily be gotten from Fig. 2. Once optimal  $\alpha^*$  is given, the value of user  $i$ 's optimal price  $p_i^*$ , optimal effort  $s_i^*$ , and corresponding data quality  $q_i(s_i^*)$  can also be gotten from Eq.(34), Eq.(5), and Eq.(1).

Secondly, we compare our optimal pricing based incentive mechanism with fixed pricing based incentive mechanism. As for fixed pricing based incentive mechanism, the IoT platform pays for all the smartphone users with the same price. The methods to calculate for optimal effort and data quality

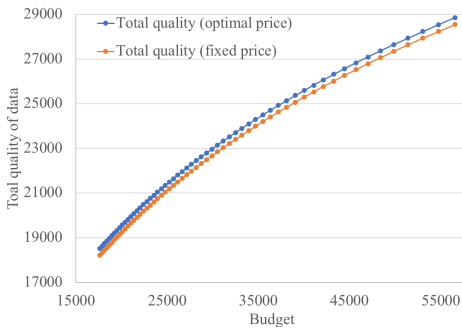


Fig. 3. Total data quality comparison.

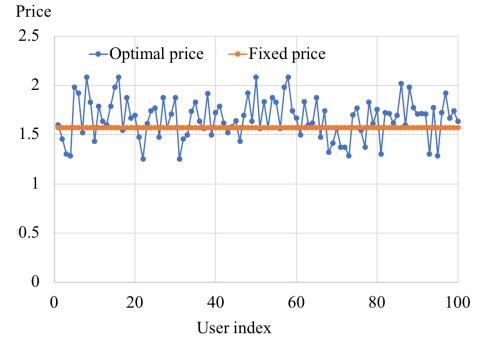


Fig. 4. Price comparison when  $B_l = 40000$ .

are the same with that of optimal pricing based incentive mechanism. Fig. 3 shows the total data quality of optimal pricing based incentive mechanism and fixed pricing based incentive mechanism. Both of them increase with the budget  $B_l$ . The total data quality of our proposed optimal pricing based incentive mechanism is slightly but constantly higher than that of the fixed pricing method. The reason behind this phenomena is that our proposed incentive mechanism sets different incentive for different smartphone users to maximize the data quality provided by smartphone user.

Fig. 4 show the comparison between the price of our proposed incentive mechanism  $p_i$  ( $\forall i \in \mathcal{N}_l$ ) and the fixed pricing incentive mechanism when the budget  $B_l$  is 40000. Please note that only data of the first 100 users out of the 1000 users are shown in the figure. It can be seen that the price of our proposed incentive mechanism varies with different smartphone users. Our optimal pricing based incentive mechanism tries to maximize the total quality of sensing data as defined in Eq. (7) by allocating the budget  $B_l$  in global manner over different users, while the fixed pricing incentive mechanism does not consider the total quality of sensing data maximization.

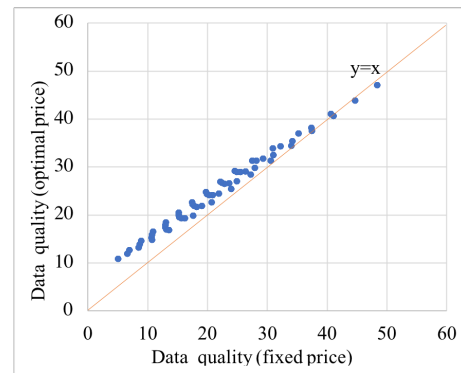


Fig. 5. Data quality comparison when  $B_l = 40000$ .

Fig. 5 shows the data quality comparison of 1000 users. It shows the different data qualities of different smartphone users are different. The horizontal axis (x axis) is the data quality of fixed price, and the vertical axis (y axis) is the data quality of optimal price. It is obvious that the 1000

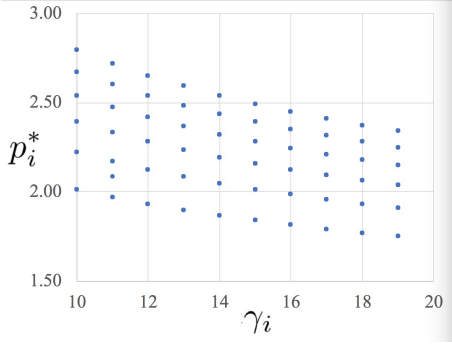


Fig. 6.  $p_i^*$  v.s.  $\gamma_i$ .

points are scattered above the function  $y = x$ , which means that the data quality of our proposed incentive mechanism is much higher than that of fixed pricing incentive mechanism.

Fig. 6 shows how optimal price  $p_i^*$  changes with user's reputation  $\gamma_i$ . It shows the optimal price are different for different smartphone users with different reputation parameters. The higher the reputation value of a smartphone user is, the lower optimal price is. As shown in problem formulation section IV, we try to maximize the total quality of data of all the users. The budget is allocated to users based on their contributed quality of data, which is a combination of users' reputation and users' effort. The lower reputation users can also get much more budget by making much higher effort. This might degrade the motivation to cooperate the system for users with high reputation, and we leave this problem in the future work.

## VII. CONCLUSION

In this paper, we propose to set the price paid to smartphone users depending on the ability of acquiring sensing data which is estimated through the reputation of users to maximize the quality of data provided by the users. We formalize the data quality optimization problem under the platform's budget constraint, and the optimum price is analyzed by using the Lambert W function and Karush-Kuhn-Tucker (KKT) conditions. Numeric analysis validated our proposed incentive mechanism is effective and showed the total data quality is maximized for the platform.

This paper only considered the data quality maximization problem for a certain location. We will consider budget allocation problem for the IoT platform for many different locations, and to maximize the total value of data of many locations. Only centralized IoT cloud server is considered in this paper, multi-access edge computing (MEC) can be used for super fast data transmission for users' data [15]. Besides, users' data protection is also an important problem [16] [17], which will be considered in the future work.

## VIII. APPENDIX

### A. Proof of Proposition 2

We define a function as follows:

$$f_i(p_i) = \gamma_i \ln\left(\frac{\gamma_i}{\lambda_i} p_i\right) \quad (19)$$

Firstly, we proof that  $f_i(p_i)$  is a concave function.

The first-order differentiation of function  $f_i(p_i)$  is as follows.

$$\frac{df_i}{dp_i} = \frac{\gamma_i}{p_i} \quad (20)$$

Then, the second-order differentiation of  $f_i(p_i)$  is as follows.

$$\frac{d^2 f_i}{dp_i^2} = -\frac{\gamma_i}{p_i^2} \quad (21)$$

Because  $\lambda_i > 0$ , it is obvious that  $\frac{d^2 f_i}{dp_i^2} < 0$ . Thus,  $f_i(p_i)$  is a concave function of  $p_i$ .

We define function the objective function of Eq.(8) as follows.

$$f(\mathbf{p}) = \sum_{i \in \mathcal{N}_i^*} \gamma_i \ln\left(\frac{\gamma_i}{\lambda_i} p_i\right) \quad (22)$$

where  $\mathbf{p} = \{p_1, p_2, \dots, p_{N_i^*}\}$  is a vector. Then we can express  $f(\mathbf{p})$  by  $f_i(p_i)$  as follows.

$$f(\mathbf{p}) = \sum_{i \in \mathcal{N}_i^*} f_i(p_i) \quad (23)$$

Since  $f(\mathbf{p})$  is nonnegative weighted sums of concave functions  $f_i(p_i)$  (see section 3.2.1 of book [18]), then  $f(\mathbf{p})$  is concave.

### B. Proof of Lemma 1

*Proof:* We define  $y = \ln nx$ , then  $x = \frac{e^y}{n}$ . By substituting  $x = \frac{e^y}{n}$  to Eq.(13), we have the transformed one as follows.

$$m = \frac{n}{e^y} \cdot \frac{1}{(y+1)} \quad (24)$$

or Eq.(24) can be expressed as follows.

$$e^{(y+1)}(y+1) = \frac{ne}{m} \quad (25)$$

According the definition of Lambert W function, we can have the solution  $y$  of Eq.(25) as follows.

$$y = W\left(\frac{ne}{m}\right) - 1 \quad (26)$$

Then  $x$  can be expressed as follows.

$$\begin{aligned} x &= \frac{e^{[W(\frac{ne}{m})-1]}}{n} \\ &= \frac{n}{e^{W(\frac{ne}{m})}} \\ &= \frac{1}{ne} \end{aligned} \quad (27)$$

According to property (1) of Lambert W function,  $x$  can also be expressed as follows.

$$\begin{aligned} x &= \frac{e^{W(\frac{ne}{m})}}{ne} \\ &= \frac{\frac{ne}{m}}{W(\frac{ne}{m})} \\ &= \frac{1}{mW(\frac{ne}{m})} \end{aligned} \quad (28)$$

### C. Proof of Proposition 3

*Proof:* The Lagrangian of the problem in Eq.(8) is as follows.

$$L(\mathbf{p}, \alpha, \beta) = -f(\mathbf{p}) + \alpha \left[ \sum_{i \in \mathcal{N}_l^*} p_i \gamma_i \ln\left(\frac{\gamma_i}{\lambda_i} p_i\right) - B_l \right] - \sum_{i \in \mathcal{N}_l^*} \beta_i p_i \quad (29)$$

where  $\alpha \geq 0$ ,  $\beta_i \geq 0$ ,  $\beta = (\beta_1, \dots, \beta_{N_l^*})$  is the Lagrangian dual variable (or Lagrangian multiplier). The Karush-Kuhn-Tucker (KKT) conditions [18] for optimality are as follows.

$$\left\{ \begin{array}{l} \text{Primal constraints:} \\ \quad \sum_{i \in \mathcal{N}_l^*} p_i - B_l \leq 0 \\ \quad p_i \geq 0 \\ \text{Dual constraints:} \\ \quad \alpha \geq 0 \\ \quad \beta_i \geq 0, \forall i \in \mathcal{N}_l^* \\ \text{Complementary slackness:} \\ \quad \alpha (\sum_{i \in \mathcal{N}_l^*} p_i \gamma_i \ln(\frac{\gamma_i}{\lambda_i} p_i) - B_l) = 0 \\ \quad \beta_i p_i = 0, \forall i \in \mathcal{N}_l^* \\ \text{Gradient of Lagrangian with respect to } p_i \text{ vanishes:} \\ \quad \nabla L(\mathbf{p}, \alpha, \beta) = 0 \end{array} \right. \quad (30)$$

$$\nabla L(\mathbf{p}, \alpha, \beta) = -\frac{\gamma_i}{p_i} + \alpha \gamma_i [\ln(\frac{\gamma_i}{\lambda_i} p_i) + 1] - \beta_i = 0 \quad (31)$$

From Eq.(31), we have

$$\frac{\gamma_i}{p_i} = \alpha \gamma_i [\ln(\frac{\gamma_i}{\lambda_i} p_i) + 1] - \beta_i \quad (32)$$

It is obvious that  $p_i > 0$ , then  $\beta_i = 0$ , we have

$$\alpha = \frac{1}{p_i^*} \cdot \frac{1}{[\ln(\frac{\gamma_i}{\lambda_i} p_i^*) + 1]} \quad (33)$$

Eq.(33) has the same structure of Eq.(13) in **Lemma 1** if we assume that  $m = \alpha$ , and  $n = \frac{\gamma_i}{\lambda_i}$ . The solution of  $p_i^*$  can be easily obtained by **Lemma 1** as follows.

$$\begin{aligned} p_i^* &= h'_i(\alpha) \\ &= \frac{\lambda_i e^{W(\frac{\gamma_i}{\lambda_i} \alpha)}}{e \gamma_i} \\ &= \frac{1}{\alpha W(\frac{\gamma_i}{\lambda_i} \alpha)} \end{aligned} \quad (34)$$

where  $h'_i(\alpha)$  is the function used to express  $p_i^*$ . By substituting property (2) of Lambert W function, the derivative of  $h'_i(\alpha)$  can be obtained as follows.

$$h'_i(\alpha) = -\frac{1}{\alpha^2 [1 + W(\frac{\gamma_i}{\lambda_i} \alpha)]} \quad (35)$$

Since  $\lambda_i > 0$ , then  $\frac{\lambda_i}{p_i} = \alpha > 0$ . By complementary slackness, we have the following conditions.

$$\sum_{i \in \mathcal{N}_l^*} p_i \gamma_i \ln\left(\frac{\gamma_i}{\lambda_i} p_i\right) - B_l = 0 \quad (36)$$

Now we formulate the dual problem.

By substituting Eq.(34) to the Lagrangian Eq.(29), we have the objective function of the dual problem as follows.

$$\begin{aligned} D(\alpha, \beta) &= L(\mathbf{p}^*, \alpha, \beta) \\ &= -f(\mathbf{p}^*) + \alpha \left[ \sum_{i \in \mathcal{N}_l^*} p_i^* \gamma_i \ln\left(\frac{\gamma_i}{\lambda_i} p_i^*\right) - B_l \right] - \sum_{i \in \mathcal{N}_l^*} \beta_i p_i^* \\ &= -\sum_{i \in \mathcal{N}_l^*} \gamma_i \ln\left(\frac{\gamma_i}{\lambda_i} h'_i(\alpha)\right) \\ &\quad + \alpha \left[ \sum_{i \in \mathcal{N}_l^*} h'_i(\alpha) \gamma_i \ln\left(\frac{\gamma_i}{\lambda_i} h'_i(\alpha)\right) - B_l \right] \end{aligned} \quad (37)$$

The formal dual problem is then defined as follows.

$$\begin{aligned} \max D(\alpha, \beta) \\ \text{s.t. } \alpha \geq 0 \\ \beta_i \geq 0, \forall i \in \mathcal{N}_l^* \end{aligned} \quad (38)$$

Since we have confirmed that  $\beta_i = 0$ , then the dual problem is transformed to the following one.

$$\begin{aligned} \max D(\alpha, 0) \\ \text{s.t. } \alpha \geq 0 \end{aligned} \quad (39)$$

The first order differentiation with respect to  $\alpha$  of  $D(\alpha, 0)$  is as follows.

$$\begin{aligned} \frac{\partial D(\alpha, 0)}{\partial \alpha} &= \sum_{i \in \mathcal{N}_l^*} \gamma_i h'_i(\alpha) \left[ \alpha - \frac{1}{h_i(\alpha)} \right] \\ &\quad + \sum_{i \in \mathcal{N}_l^*} h_i(\alpha) \gamma_i \ln\left(\frac{\gamma_i}{\lambda_i} h_i(\alpha)\right) - B_l \\ &\quad + \alpha \left[ \sum_{i \in \mathcal{N}_l^*} \left[ h'_i(\alpha) \gamma_i \ln\left(\frac{\gamma_i}{\lambda_i} h_i(\alpha)\right) \right] \right] \end{aligned} \quad (40)$$

By substituting Eq.(34) and Eq.(35), we have

$$\begin{aligned} \frac{\partial D(\alpha, 0)}{\partial \alpha} &= -B_l + \sum_{i \in \mathcal{N}_l^*} \frac{\gamma_i [W(\frac{\gamma_i}{\lambda_i} \alpha) - 1]}{\alpha [1 + W(\frac{\gamma_i}{\lambda_i} \alpha)]} \\ &\quad + \sum_{i \in \mathcal{N}_l^*} \left[ \frac{\gamma_i}{\alpha W(\frac{\gamma_i}{\lambda_i} \alpha) [1 + W(\frac{\gamma_i}{\lambda_i} \alpha)]} \ln\left(\frac{\gamma_i}{\lambda_i} \cdot \frac{1}{\alpha W(\frac{\gamma_i}{\lambda_i} \alpha)}\right) \right] \end{aligned} \quad (41)$$

We define a function  $G(\alpha)$  as follows.

$$\begin{aligned} G(\alpha) &= \sum_{i \in \mathcal{N}_l^*} \frac{\gamma_i [W(\frac{\gamma_i}{\lambda_i} \alpha) - 1]}{\alpha [1 + W(\frac{\gamma_i}{\lambda_i} \alpha)]} \\ &\quad + \sum_{i \in \mathcal{N}_l^*} \left[ \frac{\gamma_i}{\alpha W(\frac{\gamma_i}{\lambda_i} \alpha) [1 + W(\frac{\gamma_i}{\lambda_i} \alpha)]} \ln\left(\frac{\gamma_i}{\lambda_i} \cdot \frac{1}{\alpha W(\frac{\gamma_i}{\lambda_i} \alpha)}\right) \right] \end{aligned} \quad (42)$$

The point  $\alpha^*$  satisfy the following Eq.(43) is the optimal solution for the dual problem.

$$G(\alpha) = B_l \quad (43)$$

It is difficult to get the explicit solution for  $\alpha^*$ , we will get the optimal solution  $\alpha^*$  as well as  $p_i^*$  by numeric analysis.

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