

Packet-Pair Dispersion Signatures in Multihop Networks

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Abstract—Packet-pair technique is a widely used method for characterizing end-to-end network paths. A new analytical model is presented for the packet-pair based signature that accurately describes the behavior of packet-pairs in multihop network paths with multiple tight links. The relationship between the input and output gaps of packet-pairs and the corresponding distribution of end-to-end packet-pair dispersions are derived. This relationship is then used to derive the signature characterizing the path. The model is verified via OPNET® based simulations. We explore how the signature is shaped by factors such as the number of hubs, initial dispersion, and cross traffic. Path signature is an essential tool for applications that need to distinguish between different network paths. The analytical model provides deeper insights to path signatures in the presence of multiple tight links, thus enabling accurate network monitoring, problem diagnosis, and estimation of link and path parameters such as network bandwidths and link capacities.

Index Terms—Queuing models, packet-pair dispersion, path signature, network measurements.

I. INTRODUCTION

Network monitoring, tomography, and overlay based QoS provisioning are among network operations and applications that rely on the use of end-to-end path measurements to infer operational conditions of network paths. Dispersion of packet pairs or packet trains as they traverse is the basis of many inference tools used for characterization of network paths [4], [13], [16], [20], [24-26]. Packet-pair technique is used to obtain crucial network properties such as bottleneck capacity, available bandwidth and common congestion links in a wide array of network monitoring tools [1-5], [7], [9], [11-12], [14], [17], [21-23]. In general, a packet pair consists of two equal-length packets with a predefined initial time gap between them. By the time the packet-pair reaches the destination, different factors such as link characteristics and cross traffic shape the final dispersion [5]. Cross traffic refers to the regular traffic through the path but not probing packets used in the network measurement. Packet-pair technique essentially estimates the network parameters, e.g., the end-to-end network bandwidth and the bottleneck link capacity, from the relationship between the initial and final dispersions.

Packet-pair dispersions can be used to generate path signatures. Dispersion fingerprint [6] uses the Cumulative Distribution Function (CDF) of packet-pair dispersion as the path signature. Internet path signatures are distinct, in general,

and they persist over periods of time [6]. Thus the signatures are used for distinguishing network paths from each other, monitoring networks, detecting problems and anomalies, understanding network operational conditions, testing protocols for practical and real network circumstance, and for identifying whether paths share joint links. Accurate models for packet-pair technique are important for creating accurate path signatures for different scenarios; such models help enhance the accuracy and efficiency of measurement techniques for parameter estimation as well [6]. This paper develops a model for the packet-pair technique for multihop paths with multiple tight links, thus allowing for more precise capture of the stochastic characteristic of network such as cross traffic and its interaction with packet-pairs. Our work extends the existing stochastic delay model presented in [5], which describes the analytical relationship between the input and output dispersions under the assumption of a single tight link. We validate the proposed model via OPNET® simulations and discuss the accuracy of the proposed model. We then use the analytical model to investigate the effects due to initial dispersion, cross traffic and the number of the hops on end-to-end signatures. We also show how link properties such as available bandwidth, and arrival rate of cross traffic can be estimated based on link signatures. In addition, our results are directly applicable for deriving the path signature when measurements have to be done using packet pairs with variable packet gaps. This is the case with passive packet-pair techniques that rely on existing traffic stream for measure ends, thus not exhausting network resources for measurements [3].

Section II reviews the packet-pair technique and the packet-pair delay model for the single-hop case. Section III presents the queuing model to generate path signatures for the multi-hop case and determine path characterization. This model is validated through OPNET® simulations, and the impact of network and traffic characteristics on signature is evaluated in Section IV. Finally, the conclusion and future work follow in Section V.

II. PACKET-PAIR DELAY MODEL

The packet-pair technique employs two equal sized packets that a source sends to a receiver as shown in Fig. 1. The initial dispersion Δ_{in} at the sender is defined as the period between the departure of the first bit of the first probing packet P_1 and the second probing packet P_2 from the sender [6]. The path

characteristics such as link capacities and cross traffic can effect to change the dispersion between these two packets. The gap between the arrival time of the first bits of the first and second packets at the receiver is defined as the final dispersion Δ_{out} [5]. The main goal is to establish an analytical relationship between Δ_{out} and Δ_{in} . Figure 1 shows how parameters such as cross traffic and network link capacities affect packet-pair dispersion. The dispersion may change due to multiple causes. Fig. 1(b) depicts the case in which the packets travel from a high bandwidth link to a low bandwidth link, and as a consequence the final dispersion increases. In Fig. 1(c), the final dispersion increase is due to one or more cross traffic packets that get inserted between the packet pair. In Fig. 1(d), the final dispersion reduces as the packet pair goes into a busy link and one or more packets have entered the queue before the first packet in the pair [1]. The final dispersion is shaped by a mixture of these scenarios at the end of a typical path. This example represents that the network can either increase or decrease the final gap according to cross traffic rate and link capacity.

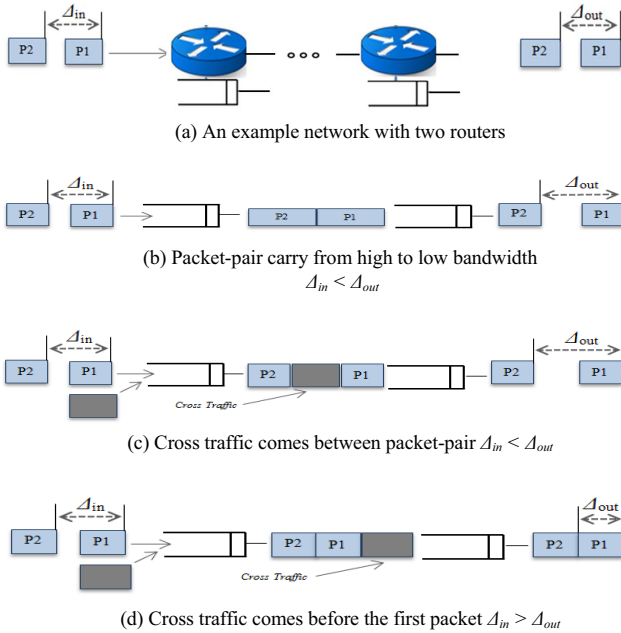


Fig. 1. Impact of traffic and network links on packet-pair dispersion

The stochastic model of packet-pair technique in [5] provides the basic relationship between Δ_{in} and Δ_{out} for a single-hop. Let q_1 and q_2 respectively be the number of packets already in the queue when the first and second probe packets enter the router queue. The service time of each packet in the pair is equal to L_P/C . Let W_1 and W_2 be the waiting times of the first and second packet. Under Poisson traffic assumption, [5] shows:

$$\Delta_{out} - \Delta_{in} = \left(W_2 + \frac{L_P}{C} \right) - \left(W_1 + \frac{L_P}{C} \right) = W_2 - W_1 \quad (1)$$

Here, the system is approximated by an M/D/1 queue model. Let $t_0=0$ be the time of the first packet arrival; the total of packets in the queue is $N_0 = q_1 + L_P/L_C + I$. C is the link capacity, λ is arrival rate and L_P and L_C respectively are the lengths of each of the packets in the pair and in cross traffic. W_2 is the waiting time of the second packet of the pair, which corresponds to the delay of a packet that reaches the M/D/1 system at $t = \Delta_{in}$.

First the state vector $\pi(t) = (\pi_0(t), \pi_1(t), \pi_2(t), \dots)$ is established as in [5], where $\pi_j(t)$ is the probability of the system retaining j packets at time t .

In contrast to [5], we use the following transition matrix P to determine $\pi_j(t)$, which has been obtained by applying a correction to that in [5] based on the proof results in [10]:

$$P = \begin{pmatrix} e^{-\lambda D} & \lambda D e^{-\lambda D} & \dots & \left(\frac{\lambda D}{j!}\right) e^{-\lambda D} & \dots \\ e^{-\lambda D} & \lambda D e^{-\lambda D} & \dots & \left(\frac{\lambda D}{j!}\right) e^{-\lambda D} & \dots \\ 0 & e^{-\lambda D} & \dots & \left(\frac{\lambda D}{(j-1)!}\right) e^{-\lambda D} & \dots \\ 0 & 0 & e^{-\lambda D} & \dots & \left(\frac{\lambda D}{(j)!}\right) e^{-\lambda D} & \dots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix} \quad (2)$$

The Cumulative Distribution Function (CDF) of the waiting time $W(\Delta_{in})$, $F_{\Delta_{in}}(x)$ in [5], corresponds to the waiting time distribution of the second packet in the pair that we will consider in our model.

III. QUEUING MODEL FOR MULTIHOP CASE

In this section, we derive the general delay model to determine relationship between dispersion of packet pair at all intermediate links in a multihop path. First, we give our rationale as to how the behavior of the packet-pair dispersion can be described and the main challenge in handling packet-pairs with multihop queuing model. Then, we develop a stochastic delay model to describe the mathematical model between initial and final dispersions of the packet pair.

$F_{\Delta_{in}}(x)$ provides a delay model to determine the dispersion of the packet pair for a specific Δ_{in} in a single link or a path with a single tight link. In multihop case, the gap between the packet-pairs entering a link is no longer a known constant value. The path signature in fact represents the net effect due to these variable packet dispersions at all the intermediate links. Therefore, a challenge in deriving a general multihop queuing model is to determine the initial dispersion of the pair entering link i , $\Delta_{in}(i)$. As in multihop model Δ_{out} from the link i is the Δ_{in} for link $i+1$, the single link model $F_{\Delta_{in}}(x) = P\{W(\Delta_{in}) < x\}$ can provide the distribution of dispersion only for a specific Δ_{in} . Simply chaining multiple instances of single link dispersions together does not work, as it requires that at the junction between two links, the packet gap be reset to a known fixed Δ_{in} . However, we have a different Δ_{in} as the input for each hop for each packet pair. To determine a general multihop model,

consider M links in an end-to-end path as in Fig. 2. Let C_i and λ_i denote the link capacity and the packet arrival rate of cross traffic respectively of the i^{th} link, $1 \leq i \leq M$. Furthermore, let $\Delta_{in}(i)$ denote the initial dispersion of a packet pair at the i^{th} link. The Cumulative Distribution Function (CDF) of the waiting time $W(\Delta_{in}(i))$, for i^{th} link, $F_{\Delta_{in}}(x, i) = P\{W(\Delta_{in}(i)) < x\}$ is as follows:

$$F_{\Delta_{in}}(x, i) = \begin{cases} \sum_{j=0}^{\lfloor \frac{x}{D} \rfloor} -N0 + K(y+D) \frac{(\lambda_i * \Delta_{in}(i))^j}{j!} e^{-(\lambda_i * \Delta_{in}(i))} & y \leq 0 \\ \sum_{j=0}^{\lfloor \frac{x}{D} \rfloor} Q_j \left[\frac{x}{D} \right] - j \frac{y^j (\lambda_i * z)}{j!} e^{-(\lambda_i * z)} & \text{otherwise} \end{cases} \quad (3)$$

Where, $y = \Delta_{in}(i) - D + (x \bmod D)$, $z = D - (x \bmod D)$, and $Q_m(t, i) := \sum_{n=0}^{m+1} \pi_n(t, i)$. We can determine $\Delta_{out}(i)$ similar to the single hop case from Eq. 1, but with W_2 and W_1 replace by $W_2(i)$ and $W_1(i)$ for the given packet pair.

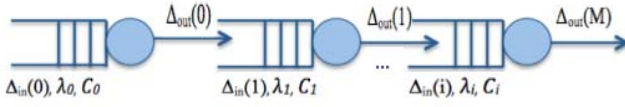


Fig. 2. Multihop queuing system model

Let $S_{\Delta_{in}}(r_i, i)$ denote the signature of link i for a given δ_{in} :

$$S_{\Delta_{in}}(r_i, i | \delta_{in}) = F_{\delta_{in}}(x, i) \quad (4)$$

where $r_i = \delta_{in}(i) + x + W_1(q_1, i)$

$S_{\Delta_{in}}$ is calculated for each given $\Delta_{in}(i) < r_{i-1}$, and r covers the range of values for $\Delta_{out}(i)$ in i^{th} link where x determines the range of distribution values for $W_2(i)$ in $F_{\Delta_{in}}(x, i)$ and we need to obtain the range of values for $\Delta_{out}(i)$ according to Eq. 4.

As $\Delta_{out}(i)$, the dispersion at the output of link i for a packet pair is the inter-arrival time $\Delta_{in}(i+1)$ for the $(i+1)^{\text{th}}$ link, we can compute the probability of each inter-arrival time $\Delta_{in}(i+1)$ at link $i+1$ by using CDF of the $\Delta_{out}(i)$ given by Eq. 4. CDF of $\Delta_{out}(i+1)$ can then be generated by multiplying $S_{\Delta_{in}}(r_i, i)$ for a specific Δ_{in} by probability of separation Δ_{in} in the current link. Probability of a specific point can be found from CDF distribution function as we consider in Eq. 5, where, $P_{\Delta_{in}}(r_i, i)$ is defined as the Cumulative Distribution Function (CDF) of the $\Delta_{out}(i)$ in the queuing model for a multihop case:

$$P_{\Delta_{in}}(r_i, i | \delta_{in} < r_{i-1}) = \begin{cases} \int_0^{\infty} S_{\Delta_{in}}(r_i, i) * (P_{\Delta_{in}}(r_i, i-1) - P_{\Delta_{in}}(r_i^-, i-1)) dr & i \geq 1 \\ S_{\Delta_{in}}(r_i, i) & i = 0 \end{cases} \quad (5)$$

For example, consider a three hop path, i.e., $M=2$ in Fig. 2. First $F_{\Delta_{in}}(x, 0)$ and r_0 are computed and $F_{\Delta_{in}}(x, 0)$ is assigned to $S_{\Delta_{in}}(r_i, i | \delta_{in})$ as well. At this step we have the CDF of $\Delta_{out}(0)$ $F_{\Delta_{in}}(x, 0)$, which also corresponds to CDF of $\Delta_{in}(1)$. To find the

delay dispersion of next link the delay dispersion for all $\Delta_{in}(1)$ that is in the range of $0 < \Delta_{in}(1) < r_0$ will be computed. Finally, we can find the delay distribution of second packet pair from Eq. 5 and then $\Delta_{out}(1)$ from Eq. 1.

Therefore, by computing $P_{\Delta_{in}}(r_i, i)$ for link i we can determine the signature of link i , and use this signature to create the next link's signature. In the other words, initially the packets are generated in the first link using the packet-pair delay model of Eq. 3 for specific $C_i, \lambda_i, \Delta_{in}$. Then, as the output packets of one link become the arrival packets for the next link, we obtain the probability of each inter-arrival time Δ_{in} of packet pair by using the previous link's delay dispersion model. Therefore, in the next link we generate the packet-pair dispersion model $S_{\Delta_{in}}(r_i, i)$ for each inter-arrival time Δ_{in} in the current link and compute the path signature according to $S_{\Delta_{in}}(r_i, i)$ and probability of each inter-arrival time Δ_{in} at link i .

A. Path Characterization

As discussed above, one of our contributions is the use of the multihop queuing model to characterize network paths. Now we show how our model can be used to determine link properties such as rate of cross traffic, utilization, and available bandwidth for each link. Initially we assume the path fingerprint is created at each point for a specific initial Δ_{in} . The signature at the end point refers to $F_{\Delta_{in}}(x, M)$. According to Eq. 3, λ_M can be obtained as follows since the other variables are known, and we have $F_{\Delta_{in}}(x, i)$ for $i=M$. For $y \leq 0$ we have:

$$F_{\Delta_{in}}(x, i) = \sum_{j=0}^{\lfloor x/D \rfloor} -N0 + K(y+D) \frac{(\lambda_i * \Delta_{in}(i))^j}{j!} e^{-(\lambda_i * \Delta_{in}(i))} \quad (6)$$

which simplifies to

$$F_{\Delta_{in}}(x, i) = e^{-(\lambda_i * \Delta_{in}(i))} * (1 + (\lambda_i * \Delta_{in}(i)) + (\lambda_i * \Delta_{in}(i))^2 + \dots)$$

Since $|(\lambda_i * \Delta_{in}(i))| < 1$, this is approximated by:

$$F_{\Delta_{in}}(x, i) = e^{-(\lambda_i * \Delta_{in}(i))} * \frac{1}{1 - (\lambda_i * \Delta_{in}(i))} \quad (7)$$

Hence, the rate of cross traffic λ_M in link M can be obtained from Eq. 7. A similar process applies to find λ_M for $y > 0$. The utilization and available bandwidth of link M can then be determined from $A = C_i (1 - u_i)$ since the capacities of the links are assumed to be known. This procedure can be extended to find other link properties as well by using the corresponding path fingerprint for each link.

IV. MODEL VALIDATION

In this section we validate our stochastic model via OPNET[®] simulations and explore how the signature is shaped by factors such as the number of the hops, link capacity, initial dispersion and cross traffic arrival rate.

We perform simulation for two-hop, four-hop and eight-hop networks, connected via 100 Mb/s links. The cross traffic on each node is set at 20% and 60% of link utilization respectively for low and heavy Poisson cross traffic conditions. Non-persistent cross traffic [2] as shown in Fig. 3 is used in which packets exit one hop after entering the path. Packet pairs are generated as the two back-to-back 1500-byte packets with 120 μ s initial dispersion. We normalized the dispersion at the end point by the initial dispersion and determine Δ_{out}/Δ_{in} as the normalized dispersions.

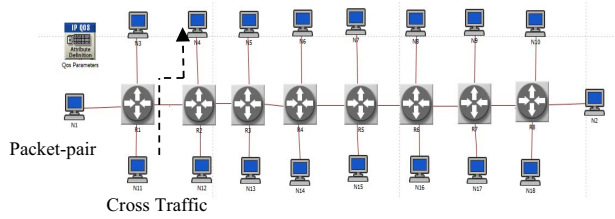


Fig. 3. Multi-hop Model in OPNET[®] simulation

Figure 4 and 5 shows path signatures based on CDF of packet-pair dispersion obtained with OPNET[®] simulation and our model for low and heavy cross traffic. The simulation results agree very closely with the stochastic model in Eq. 5. In addition, we investigated the effect of number of hops in estimating the path signatures. When the number of hops is increased from 2 to 8 the stochastic model continues to perform quite accurately for heavy cross traffic utilizations. The maximum error of CDF between OPNET[®] simulation and stochastic model in heavy cross traffic condition is 0.051 when path signature is generated for eight-hop paths indicating close agreement between stochastic model and simulation results.

The other evaluation part for each signature is investigating available bandwidth estimation of tight link based on our analysis. The available bandwidth estimation is computed from Eq. 7 and shown in Tables I and II for the first scenario. The estimation error shows that our model is able to determine the available bandwidth for the tight link fairly closely; however, the result indicates that the error increases as the number of the hops grows. Moreover, it is clear that having heavy cross traffic affects the accuracy of signatures as well as available bandwidth estimation. Thus the error of estimation for heavy cross traffic is more than at low cross traffic.

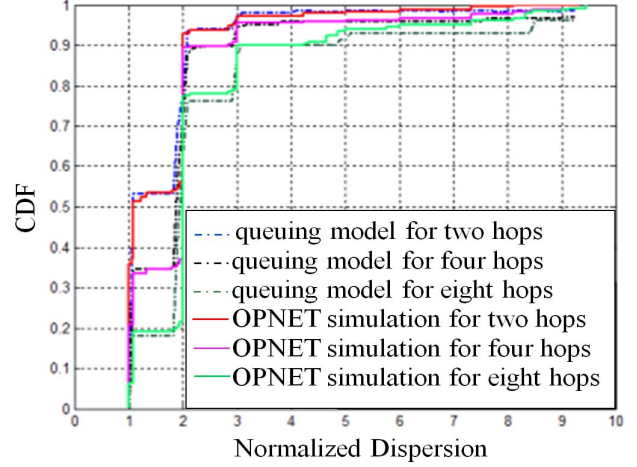


Fig. 4. Queuing model and OPNET[®] simulation results for low cross traffic and back-to-back packet pair

TABLE I. AVAILABLE BANDWIDTH FOR LOW CROSS TRAFFIC

Available Bandwidth	Actual value (Mb/s)	Estimated value (Mb/s)	Estimation error (Mb/s)
Two hops model	80	79.7	0.3
Four hops model	79.8	79	0.8
Eight hops model	80.5	79.1	1.4

Next part of the experiment evaluates our model by injecting packet pairs with 240 μ s initial dispersion. In this case we investigate the effect of initial dispersion on path signature and the accuracy of analytical model. As in the previous experiments, we can see in Fig. 6 and 7 that the model follows OPNET[®] simulations well in both low and heavy cross traffic conditions. The maximum errors in these results are 0.036 and 0.058 for low and high cross traffic respectively, when dispersion CDF generated for eight hops model.

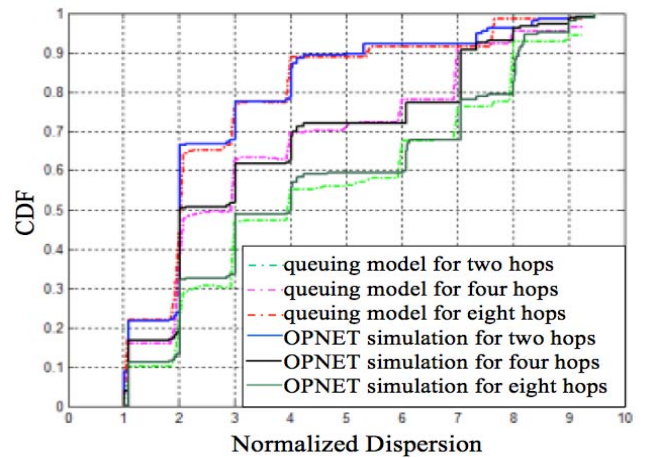
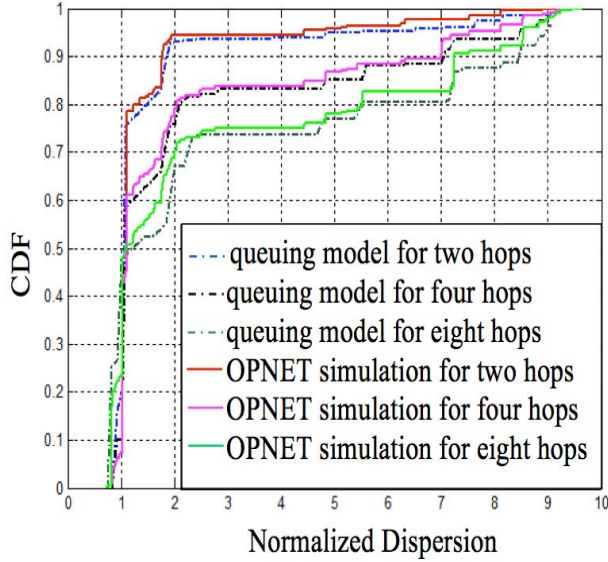


Fig. 5. Queuing model and OPNET[®] simulation results in heavy cross traffic

TABLE II. AVAILABLE BANDWIDTH FOR HEAVY CROSS

Available Bandwidth	Actual value (Mb/s)	Estimated value (Mb/s)	Estimation error (Mb/s)
Two hops model	40.1	39.5	0.6
Four hops model	40.3	39	1.3
Eight hops model	39.9	38.1	1.8

Fig. 6. Queuing model and OPNET® simulation results for low cross traffic and 240 μ s initial dispersion

Next we evaluate the impact of link capacity values on the shape of path signature. Figure 8 shows the path signatures based on different link capacities and low cross traffic. In this case we perform simulations for paths with 2 and 4 hops. The capacities of links are 100, 80, 60 and 40 Mb/s in a sequence. For two hops case links capacities are set to 100 and 80 Mb/s respectively. The path signatures still have reasonable accuracy, with maximum errors of 0.040 and 0.049 for two and four hops cases respectively. The results presented above as well as additional simulations we performed show that the maximum errors are reasonable.

The dispersion CDFs are distinguished and continuing for a given path and shape according to link characteristics such as cross traffic, initial dispersion and link capacity, and agrees with the observations in [6]. Therefore, finding an accurate model to generate path signatures for a multi hop model is important to facilitate estimation of other useful properties of the path, such as utilization, bandwidth and cross traffic rate as described in the previous section. We evaluate path characteristics, as described in the previous section, for different link capacities of the path. A path with 4 hops and the links of capacities as 100, 80, 60 and 40 Mb/s in sequence. As described, in the first step, the arrival rate of cross traffic is determined from Eq. 6, and then the utilization and available

bandwidth are computed for each link. The results in Table III show the estimated utilization. The worst estimation error for available bandwidth is not more than 1.9%, which occurs on the last link.

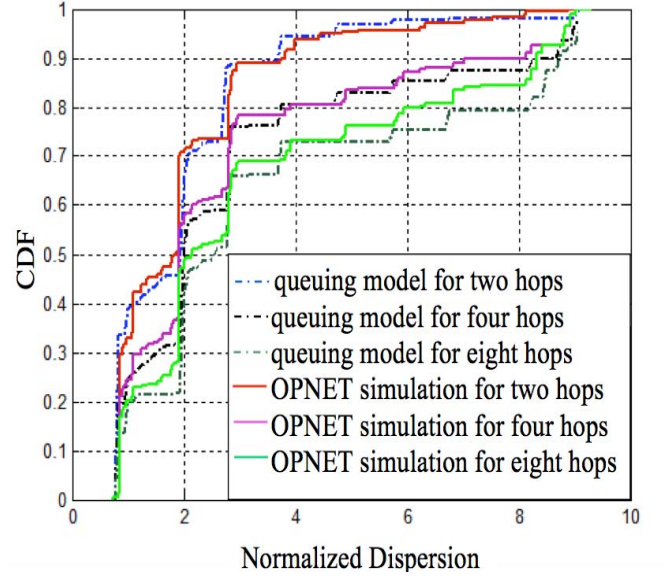
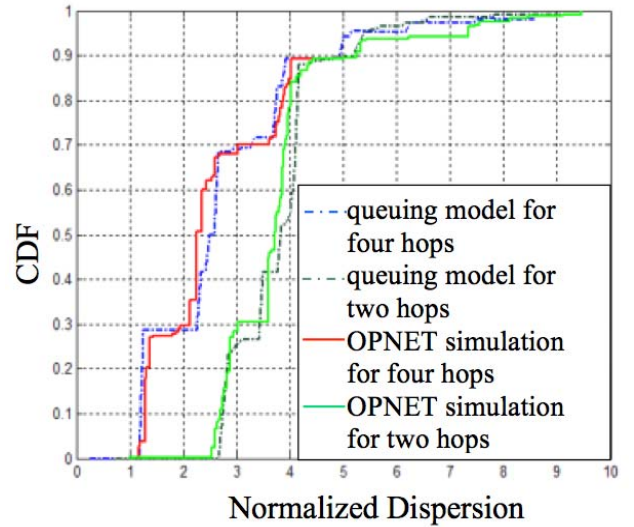
Fig. 7. Queuing model and OPNET® simulation results for heavy cross traffic and 240 μ s initial dispersion

Fig. 8. Queuing model and OPNET® simulation results for different link's capacity in low cross traffic

TABLE III. UTILIZATION FOR DIFFERENT LINKS IN LOW CROSS TRAFFIC

Utilization	Actual value %	Estimated value %	Estimated error %
Link 1	21.2%	20.7%	0.9%
Link 2	20.1%	21.3%	1.2%
Link 3	19.9%	20.6%	1.7%
Link 4	21.5%	19.6%	1.9%

From the overall results we can see that the proposed model can assist to determine the path and link properties in the network; however, we have made the assumption that packets arrive according to a Poisson process with rate λ and provide for an M/D/1 based queuing model. In the last part of our evaluation, we run experiments with another type of distribution for cross traffic on OPNET[®] and compare it with the path signature based on our queuing model. This result checks the sensitivity and accuracy of model when cross traffic does not follow a Poisson process. In this case we repeat the first scenario for two and four hops with Pareto arrival process. In general, the Pareto distribution is applied to model self-similar arrival in packet traffic for simulation tools. The results are shown in Fig. 9 and 10 for low and heavy utilizations. The maximum error between OPNET[®] simulation with Pareto distribution and stochastic model in low cross traffic condition is 0.09 when path signature generates for four-hop model, and for heavy cross traffic rate maximum error is 0.19. Practically, the shape of signature from our model follows the OPNET[®] simulation in this case as well; however, the error increases from 0.03 to 0.09 for low cross traffic rate and from 0.051 to 0.19 for heavy cross traffic. In addition, the path signatures show the accuracy of our model reduces as the number of the hops increase. The estimation error in Tables IV and V shows that our model has 2.9Mb/s estimation error in the worst case for heavy cross traffic and four-hop model. As the results demonstrate, it is clear that having the other type of arrival cross traffic model cause the reduction in accuracy to provide path properties; however, the estimation error is fairly acceptable in our experiment. Moreover, it is predictable that having more number of hops could affect the accuracy in this condition as well.

In summary, the proposed model can provide the path and link properties in the network based on Poisson model for cross traffic accurately and indicate that the our model can determine appropriate information of the path based on path signature.

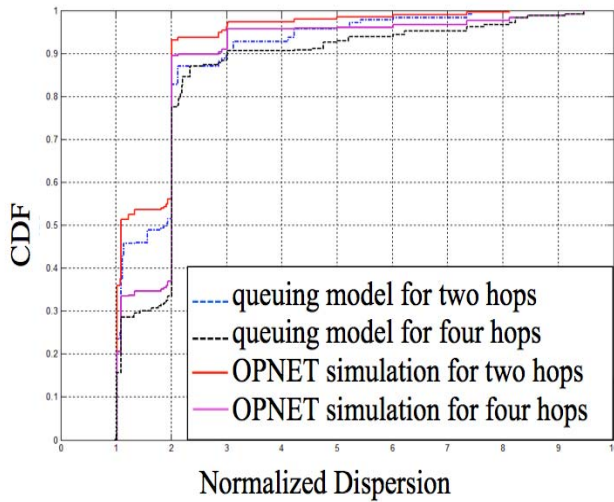


Fig. 9. Queuing model and OPNET[®] simulation results for Pareto distribution and low cross traffic

TABLE IV. AVAILABLE BANDWIDTH FOR LOW CROSS TRAFFIC AND PARETO DISTRIBUTION

Available Bandwidth	Actual value (Mb/s)	Estimated value (Mb/s)	Estimation error (Mb/s)
Two hop model	80.6	79.7	0.9
Four hop model	80.6	79	1.6

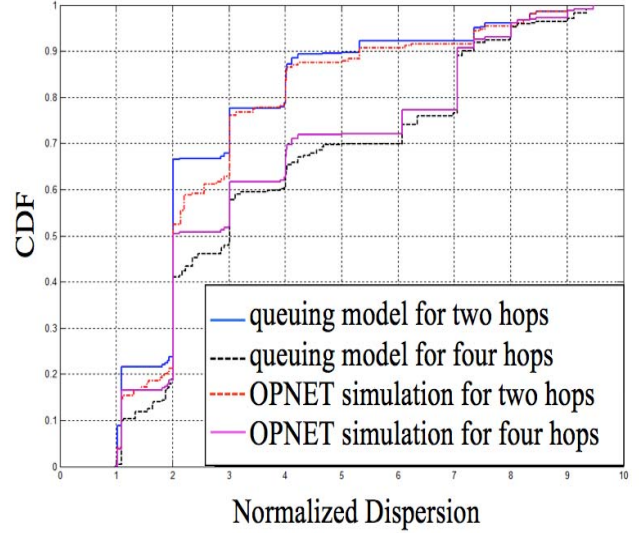


Fig. 10. Queuing model and OPNET[®] simulation results for Pareto distribution and Heavy traffic

TABLE V. AVAILABLE BANDWIDTH FOR HEAVY CROSS TRAFFIC AND PARETO DISTRIBUTION

Available Bandwidth	Actual value (Mb/s)	Estimated value (Mb/s)	Estimation error (Mb/s)
Two hop model	41.3	39.5	1.8
Four hop model	41.9	39	2.9

V. CONCLUSION

A mathematical model to generate accurate signatures for multihop paths, without the limitation of a single tight link between initial and final dispersions of packet pairs was presented. The analytical model was verified using simulations, and was used to evaluate the impact of factors such as the number of hops, initial dispersion, link capacities and cross traffic, that affect the shape of the signature. Results from the proposed model agree closely with OPNET[®] simulations. As the path signatures can provide other properties of a path, such as estimate for available bandwidth, utilization, and bottleneck capacity of the path, the model can be used to determine useful path properties from the path signatures. Development of the multi hop model for general conditions and a G/G/1 based queuing model as well as validating the model with real-world network setup remain as future work.

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