

The Improvement of Positioning Accuracy by the Advanced Gaussian Process Method

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Abstract—Location data is the most widely used context data in ubiquitous and mobile computing applications. To support locating applications in indoor positioning, many systems with distinct deployment costs and positioning accuracies have been manipulated recently. The most useful method is focused on the received signal strength and provides a set of signal transmission access points. However, compiling a manual measuring RSS fingerprint database involves high cost, which is impractical in an on-line prediction environment. In this study, the system is relied on the Gaussian Process method, which is a non-parametric model can be characterized by the mean and covariance functions. Besides, the Naive Bayes method is also used to the verified and simplified computation of the precise prediction. According to our experiments, the authors proposed several experiments on the simulation and real environment in Tianjin University. The experiments are beholding on the distinct data size, different kernels and accuracies testing. All the results show that our proposed method not only can retain the positioning accuracy, but also can save the computation time in the location prediction.

Index Terms—location estimation; RSS fingerprinting; Gaussian Process Regression; Naive Bayes

I. INTRODUCTION

In the future social application, accurate, reliable and real-time indoor positioning and position-based protocols and services are required in the future social application. A positioning system can determine its position with the mobile device, and predicate the user's location to supply the position-based services such as tracking, advertising or monitoring, etc. Generally, indoor localization problem is used in the low-cost, standard and beacon deployed environment, like the strength databases of received signal. To avoid on the resources cost of deploying infrastructure and data collecting procedure, our goal is focused on the design and implement data fusion methods by the using existing infrastructure.

Generally, received signal strength (RSS) based fingerprinting is on the idea that a unique set of signal values is collected in the unique location. When the system boots up, mobile devices receive the system's unique RSS value, and they search the fingerprinting database and identify the entry that is most similar to the system's unique RSS value as the estimated location. The main problem of a general RSS fingerprinting system is that the real RSS value is easily infected by the object, human and multipath fading effects. In other words, the RSS

fingerprint obtained at different times need not match the previous fingerprint stored in the database, leading to incorrect estimation results.

Previous studies show that fingerprinting method can achieve meter-level accuracy if the RSS signature map is updated frequently. However, due to the nature infecting of radio channel and variations in the environment, the mapping RSS signatures have to update regularly. That is, the time and effort are required to create the RSS signature map during the offline phase, and this would be the main drawbacks of fingerprinting based method. Besides, the challenges in deploying a scalable fingerprinting systems and the user facing limits are also the barriers to reach the required accuracy of commercial applications. There are many researches prompted in the elimination of the gathering RSS signatures. However, although the mapping efforts of the RSS signatures have been reduced, the method of median error is not good enough for the practical positioning application. In our study, the authors present a probabilistic infrastructure by using the sparse training data in fingerprinting building problem. Particularity, the effort and cost on the calibration of the RSS signature map can be reduced by the modeling of signal strength as the advanced Gaussian Process (Gaussian Process Plus, GPP for short), and the troublesome of RSS signature map also can be idealized at the online phase.

In the application of regression and classification, supervised learning is an important component in the statistics and machine learning for analysis of data sets, or more complex problem. For this purpose, traditionally parametric models have been developed. There have some possible advantages in the ease of interpretability. However, the simple parametric models have not much expressive power to handle the complex data sets.

A Gaussian Process (GP) is a non-parametric model, the mean function and covariance matrix are used as the kernel functions. Several hyper parameters can be used to estimate the training measurements in GP. In our study, the authors use the conditional probability of Gaussian and the prior distribution of the regression. Our proposed method also predicts the received signal strength at locations with no prior measurements, and the Naive Bayes is used to derive the obtained conditional probability of the final result derived from the GP. The whole computation process we named it as the GPRP. The main advantage of GP predictor is on the addition to the mean

estimate of the RSS, and the variance of estimation is also creating by the providing of the uncertainty of the processing. The aforementioned procedures can be done in the offline once a time.

Yiu et. al consider Gaussian Process for RSSI prediction of the indoor location problem. Partial measurements were used from the interesting area firstly. Then, they used the Firefly algorithm to train and categorize the prior results derived from the GPR. The trained results were used to build the refined fingerprinting database for the entire area. The experiment reveal that GPR model really can achieve the good result in predicting the RSS of area with no prior measures. However, according to Yiu's idea [3], the extension of GPR model is restricted by the rebuilding fingerprinting database, besides, the computation for the GPR models is still limited in their study. Thus, for the purpose to prove our idea, the authors proposed the Gaussian Process Regression Plus (GPRP) method to solve the above issues, and testified the experiments in the simulated and real environment.

The remainder of our manuscript is organized as follows. Section II reviews related studies that have investigated the properties of RSS indication for indoor positioning systems. Section III provides details of the measurement system used in the current study and describes the data analysis method. Section IV presents the study experiments and discussion, and Section V offers conclusions and recommendations for future research.

II. PRELIMINARIES

A. RSS fingerprint

The RSS properties, which facilitate location fingerprinting, have been determined by many studies on indoor positioning systems. In, it was observed that the user's orientation could cause a variation of up to 5dBm in the RSSI level. At each location, different orientations of the users and mobile devices with respect to the transmitter could cause the mean RSS value to differ. The modelling of RSS-based location fingerprinting is essential for location determination algorithms; examples of RSS-based location fingerprinting models are the probabilistic approach model and preliminary analytical model. In and, Gaussian or lognormal distributions were used to model the randomness of the RSS. For example, [3] summarized in their large-scale measurement that most RSS histograms could be fitted very well with Gaussian distribution but that few histograms could be fitted with bimodal Gaussian distributions.

Current signal-based RSS location systems have two problems: considerable manual calibration effort is required to construct a radio map in the offline training phase and the positioning accuracy changes with the environmental dynamics. In, three dynamic factors observed to change frequently over time in the environment were proposed—presence of people, relative humidity, and movements. These factors easily affect the radio signal propagating from APs to mobile devices and are responsible for changes in the positioning accuracy. In Figure 1, results of experiments conducted in a cafeteria show that an RSS distribution at a fixed location with a specified AP is not constant in time. The RSS values calibrated previously in the radio map

may be outdated, which degrades positioning accuracy because of the presence of people.

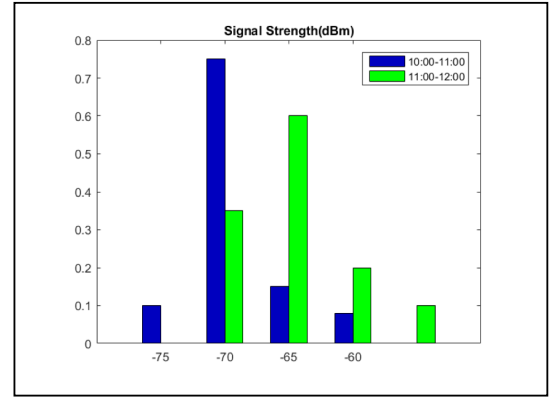


Fig. 1. RSS distributions during different time in the cafeteria

Traditionally, average RSS has been considered to be lognormal distributed according to a large-scale fading model. It is generally predictable and follows several standardized path loss models.

The wall attenuation factor (WAF) model is useful for describing the slow-fading phenomenon and the attenuation in signal propagation in indoor environments. In this model, the attenuation factor is used to predict the signal propagation behavior when walls are the main obstacle. The following equation shows how attenuation influences RSS:

$$P(d) = P(d_o) - \alpha \times \log(|d - d_o|) + \text{obstacleMat}(d), \quad (1)$$

where α indicates the rate of increase in the signal attenuation with the propagation distance, $P(d_o)$ is the RSS at a distance of reference point d_o , and d is the distance between the transmitter and the receiver. Furthermore, nW is the number of obstacles (walls) between the transmitter and the receiver; the WAF is considered as the attenuation value resulting from the obstacles. In this equation, if nW is greater than a certain constant C , this value is considered to represent the number of walls at which the attenuation factor stops influencing the signal. We can then use the constant value instead of nW .

When we select a subset of APs satisfying a certain property in our alternative set of fingerprint definitions, we should retrieve the corresponding position from the offline fingerprinting database by using the location estimation algorithm. Generally, there are several methods for determining the nearest neighbor's location.

B. Gaussian Process

Gaussian process regression is a new machine learning method based on the context of Bayesian theory and statistical learning theory^[6]. It provides a flexible framework for probabilistic regression and is widely used to solve high-dimension, small-samples or nonlinear regression problems. From the view of function space, Gaussian Process (GP) defines a distribution over functions, GPs are the extension of

multivariate Gaussian to infinite-sized vector of real-valued variables^[4,5,6]. A Gaussian Process is fully specified by a mean function and a covariance function, like

$$\begin{cases} m(x)=E[f(x)] \\ k(x|x')=E[(f(x)-m(x))(f(x')-m(x')))] \end{cases} \quad (2)$$

Where $x, x' \in R$ are random variable, the GP equation is $f(x) \sim gp(m(x), k(x, x'))$. For simplification, people usually set the mean function as zero in the data Pre-processing stage.

Since the key assumption in GP modeling is the data can be represented as a sample from a multivariate Gaussian distribution, we can infer that

$$\begin{bmatrix} x_* \\ x_* \end{bmatrix} \sim N\left(0, \begin{bmatrix} K & K^T \\ K^* & K^{**} \end{bmatrix}\right) \quad (3)$$

where T indicates matrix transposition. As we know, the function probability follows a Gaussian distribution, the conditional probability of $x_*|x$ can be computed as

$$p(x_*|x) \sim N(K_*K^{-1}y, K^{**}K^{-1}K^*) \quad (4)$$

the best estimation for x_* is the mean of this distribution $\bar{x}_* = K_*K^{-1}x$, and the uncertainty in the estimation is the variance $var(x_*) = K^{**} - K_*K^{-1}K^*$. From the above description, we can know GPR is suitable for predicting unknown data with limited known data. GPR is also the central idea we use in the paper.

Some works have been done to use Gaussian process to help generating the fingerprint database in the offline phase. Ferris et al. demonstrate how to use Gaussian process to generate likelihoods at locations for which no calibration data is available, prove that Gaussian regression can be applied successfully to various localization problems. Atia et al. consider to use GPR (Gaussian Regression Process) technique to bridge GPS-outage, finally get 80% improvement in position RMSE. Richter et al. [2]. analyze different GPR models for WLAN fingerprinting, provide some effective tips in using GPR. Cho et al. [4]. create GPR based radio-map construction method to help surveying data, which can obtain not only high accuracy but also high availability even though realistic data is rare. All these studies prove that GPR can help to improve positioning accuracy and release labor.

III. SYSTEM DESIGN

According to our previous statements, Yiu used the trained Gaussian process model to generate an estimation of signature map for entire area. For the purpose to optimize hyperparameters in GPR, they proposed the method tried to rebuild the signature map for the experiment, and conservative computation also restricted the flexibility of Gaussian model. Thus, in our study, we focus on the improvement of the GPR system and derive the system with Naive Bayes to reduce the computation complexity on the indoor positioning system.

Firstly, we create the discrete RSS fingerprinting database in the testing area. Accordingly, we derive several GP models for the system computation. The different GP models have distinct distribution functions and hyperparameters. Besides, the method of Naive Bayes is employed to select all the predicted locations from the GP models.

A. The Method of Gaussian Process Regression Plus

In our study, we need to build the offline RSS fingerprinting database firstly. Thus, we suppose the testing area has M reference points in the locations $L_r (r \in \{1, 2, \dots, M\})$.

In the sampling phase, we collect the RSS value at r th reference point of N beacons (AP) from 1 to j th times; that is

$$AP_r = \{ap_{r,1}, ap_{r,2}, \dots, ap_{r,N}\},$$

where

$$ap_{r,1} = Mo(\{ap_{r,1}^1, ap_{r,1}^2, \dots, ap_{r,1}^j\}),$$

note that $Mo(\cdot)$ is the mod function, which is used to derive the remainder of division. $ap_{r,N}^j$ is denoted as the odd moment data at r th reference point of NAP from 1 to j th times, on the contrary, AP_r is the filtering set of signal strength received at the r th reference point.

When we try to predict the unknown position at the input of RSS values, we obtain a series of RSS values AP_* ;

$$AP_* = \{ap_{*,1}, ap_{*,2}, \dots, ap_{*,N}\}.$$

We can coincide with the regression problem as $L(AP_*) = f(AP_*) + v$.

The definition of latent function $f(\cdot)$, which describes the relationship between RSS and the spatial coordinates. v is donated as the physical noise, the value follows Gaussian distribution.

According to the definition, GP is defined as the function of mean and covariance, which is used to represent the principle characteristic and structure of the model. Besides, RSS value in location positioning can be derived to the absolute distance, in our study, the authors derive the log-distance path-loss model as the mean function, that is

$$m(AP_*) = \frac{\sum_{j=1}^N \alpha_j \times (C_j - \zeta_j \times \log(\|AP_{*,j} - AP_{*,j}^S\|) + \epsilon_j)}{\sum_{j=1}^N \alpha_j} \quad (5)$$

where C_j is a constant value; $AP_{*,j}$ is the online collected RSS value of unknown position $*$ at j th AP, $AP_{*,j}^S$ is the RSS value of j th AP at the source of unknown position $*$. α_j represents the weight of $AP_{*,j}$, which indicates the trustable degree of AP. ζ_j represents the path-loss exponent of j th AP, which is computed as the noise degree of j th AP in our study, this value follows

Gaussian distribution $N(0, \sigma^2)$, σ^i is the covariance of noises in our experiment. Accordingly, we also define the covariance function as follows,

$$k(AP_*, AP_r) = \exp\left(\frac{\sum_{j=1}^N \beta_j \|AP_{*j} - AP_{rj}\|}{\sum_{j=1}^N \beta_j}\right), \quad (6)$$

where β_j is computed as the weight of AP_j . In our study, for the purpose to reduce the computation complexity, we derive the mod function for instead, that is, we use AP_r^m as an instead of AP_r in Eq. (6).

Accordingly, the objective of our system is to predict of the received signal strength $f_n(AP_*)$ for all 2-dimensional inputs x and y . To achieve this purpose, the system models are defined as

$$f_n(AP_*) \sim GP(m(AP_*), k(AP_*, AP_r)).$$

That is, the GP models for the indoor positioning are divided into part, one is based on the x and y coordinates of all training locations respectively. Thus, let $X_n \triangleq [x_1, x_2, \dots, x_S]$ be a set of training samples where S represents the number of training coordinate data, then

$$S_x = \{(x_1, y_n(x_1)), (x_2, y_n(x_2)), \dots, (x_S, y_n(x_S))\}$$

and,

$$S_y = \{(x_n(y_1), y_1), (x_n(y_2), y_2), \dots, (x_n(y_S), y_S)\}$$

are the S -dimensional column vector containing the received signal strength measurements from all training locations.

Accordingly, we defined the GP model to predict location as follows,

$$f_x(AP_*) = \frac{\sum_{j=1}^N \alpha_j \times (C_j - \zeta_j \times \log(\|AP_{*j} - AP_{*jS}\|) + \epsilon_j)}{\sum_{j=1}^N \alpha_j} + \exp\left(\frac{\sum_{j=1}^N \beta_j \|AP_{*j} - AP_{*j}\|}{\sum_{j=1}^N \beta_j}\right) \quad (7)$$

$x \in \{1, 2, \dots, w\}$

and

$$f_y(AP_*) = \frac{\sum_{j=1}^N \alpha_j \times (C_j - \zeta_j \times \log(\|AP_{*j} - AP_{*jS}\|) + \epsilon_j)}{\sum_{j=1}^N \alpha_j} + \exp\left(\frac{\sum_{j=1}^N \beta_j \|AP_{*j} - AP_{*j}\|}{\sum_{j=1}^N \beta_j}\right) \quad (8)$$

$y \in \{1, 2, \dots, h\}$

Note that, we have hyperparameters vector $\theta \triangleq [C, \alpha, \epsilon, \beta]$ to optimize our GP model, different parameters would influence the estimated results. In our experiments, we will train the hyperparameters by the training data S_x and S_y .

B. Naive-bayesian Location Model

According to previous statements, our proposed system derives the positions x and y with S_x and S_y individually. To

combine the previous results, in our study, we employ the Naive-bayesian model. Bayes' Theorem is a simple mathematical formula used for calculating conditional probabilities^[6]. Suppose E is a random experiment, $B, A_1, A_2, \dots, A_n$ are events in E , and meet the following conditions:

- (1) $P(A_i) > 0, i=1, 2, \dots, n$,
- (2) Events $A_1, A_2, \dots, A_n$ are partition by the sample space, their values are independence to each other,

$$P(B) > 0.$$

$$(3) P(A_i|B) = \frac{P(A_i B)}{P(B)} = \frac{P(A_i)P(B|A_i)}{\sum_{j=1}^n P(A_j)P(B|A_j)}, i=1, 2, \dots, n.$$

This equation can be proved by the conditional probability theorem and total probability theorem, which named after Thomas Bayes.

Bayesian Decision Theory is built on Bayesian probabilities, which allow us to assert a prior belief in a data point coming from a certain class. This theory is used as a classifier like naive Bayesian classifier, which can classify rapidly and accurately. According to our proposed method, RSS values transmitting form beacons are all independent, and our proposed GPR model are derived some predicated candidate sets of S_x and S_y . For the purpose to conclude the possible sets, naive Bayesian positioning method is used to combine and derive the best results. The naive Bayesian positioning method is listed as follows,

- (1) Sample space divided by S_x and S_y , and events in the sample spaces are represented as $L_i = (x_i, y_i)$, which is the place derived from the proposed GPR method.
- (2) Compute prior probability $P(L_i)$ as
- (3) $P(AP_*|L_i) = P(AP_1, AP_2, \dots, AP_n|L_i) = \prod_{j=1}^n P(AP_j|L_i)$ (9)
- (4) Compute posterior probability
- (5) $P(L_i|AP_*) = \frac{P(AP_*|L_i)}{P(AP_*)} = \frac{P(AP_*|L_i)P(L_i)}{\sum_{j=1}^n P(AP_*|L_j)P(L_j)}$.

Accordingly, the final estimation location is defined as

$$L_i: P(L_i|AP_*) = \max(P(L_i|AP_*))$$

and if $P(L_i|AP_*)$ is equals to zero, x and y would be selected as follows,

$$P(AP_*|x_i) = P(AP_1, AP_2, \dots, AP_n|x_i) = \prod_{j=1}^n P(AP_j|x_i) \quad (11)$$

and we derived

$$P(x_i|AP_*) = \frac{P(AP_*|x_i)}{P(AP_*)} = \frac{P(AP_*|x_i)P(x_i)}{\sum_{j=1}^n P(AP_*|x_j)P(x_j)} \quad (12)$$

then

$$P(y_i|AP_*) = \frac{P(AP_*|y_i)}{P(AP_*)} = \frac{P(AP_*|y_i)P(y_i)}{\sum_{j=1}^n P(AP_*|y_j)P(y_j)} \quad (13)$$

Compute the final location and error between estimate position as follows,

$$P(x_i|AP^*)=\max(P(x_i|AP^*)), P(y_i|AP^*)=\max(P(y_i|AP^*)),$$

and

$$P(y_i|AP^*)=\max(P(x_i|AP^*)), P(y_i|AP^*)=\max(P(y_i|AP^*)).$$

Then we defined real location is TL_i and $FL_i=\{(x_i, y_i)\}$, the root square mean error (RMSE) is derived as

$$Error=\sqrt{\frac{\sum_{i=1}^N(TL_i-FL_i)^2}{N}}. \quad (14)$$

IV. EXPERIMENTS AND DISCUSSION

A. Experiment Environment Initialization

In this section, the author setup up the experiments into two environments, one is the simulation experiments, which is programmed by the Matlab2015. The other environment is the physical experiment; which area includes four beacons deployed on the corner in a square area of size 30mx30m. In the simulation experiment, the signal is smoothing, thus, RSS signal energy declines gradually from the source, so the received energy can be predicted according to the distance from source easily. On the contrary, the physical experiment was performed on the second floor in 55th office building of Tianjin University. It is approximately a 7m by 15m area (please review the Figure 4). This area is a typical building hall with some pillar on the corner, relatively open in the center and randomly people can walk around. For the purpose to testify the signal of Bluetooth, four Bluetooth transmitters (SmartBeacon-4AA) on the corner of the area are provided to full coverage of the testing area.

Area measurements are divided into 54 locations of the testing area. Locations are marked as black dots in Figure 4. Four Bluetooth transmitters are placed and marked as red dots. For the purpose to receive the online RSS data, our experiment employs the Samsung Note3 as the data collector in the testing area. Each Bluetooth transmitter is designed to transmit 100 signals per second, and our system collect 20 samples in each location, 15 signals are for system training, 5 signals are for localization testing. In addition, we also apply our GPRP method with the mod function to avoid the potential over-fitting problem by the single signal measurement.

B. GPRP robustness on location accuracy testing

According to Eq. (1), WAF is defined as the obstacle factor, which is used to represent the signal strength receiving with obstacles effecting. To test the robustness, the validation is performed according the environment changed. Thus, the WAF value are changed on the off-line training and on-line testing. We defined four different environments in Table 1. These four

environments are represented the same off-line training and dynamic on-line testing with light, light, middle, heavy and super heavy crowds. The experiment result is shown in Fig.5. Fig.5 shows the GPRP method can adjust to the dynamic environments in the acceptable accuracy. For example, we can observe the accuracy drops less to 10%, even though in the light crowds training to heavy crowds testing.

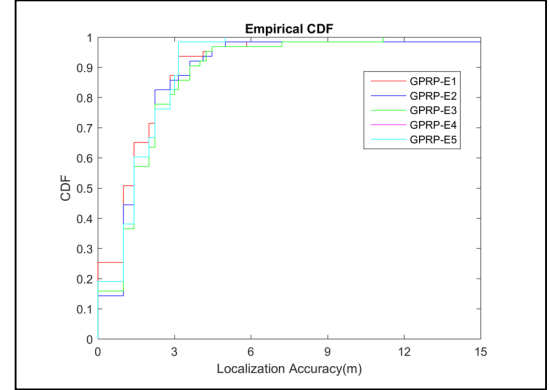


Fig.5. CDF of localization error for different obstacle environments

TABLE 1. ROBUSTNESS TESTING FOR FOUR DIFFERENT ENVIRONMENT

ID for Testing Environment	WAF for off-line training	WAF for on-line testing
E1	0.2	0.2
E2	0.2	0.3
E3	0.2	0.5
E4	0.2	1
E5	0.2	2

C. Localization Accuracy on Different Environment

In these experiments, we evaluate our system performance on the two different environments, one is based on the simulated environment, the other is based on the real environment in the Tianjin University, second floor of 55th building. For all the experiments, we adopt 30% data for training and 70% data for testing.

In this experiment, the authors compare three methods, our proposed GPRP method, Yiu's method in [3] and FP method for the general fingerprinting database. FP method is the basic method for the RSS location system, which is needed to build the RSS mapping database of the whole area. The computation time and database preparation are both the bottleneck of the system execution. On the contrary, Yiu's method adopts the flexibility of Gaussian Processing, but restores the values to the fingerprinting database. In Fig.6, the results show that the performance of our method is almost the same to the FP method, however, we can save half of the computation time to the FP method. The advantage is based on the computation of deriving method to the look-up table. Besides, Yiu's method has the partial advantage of flexibility, the computation is the worse one in the comparison. The average errors are 1.82 m, 1.46 m and 4.88 m to our proposed GPRP method, FP method and Yiu's method. Further, the cumulative accuracies in 3 m are 0.87, 0.90

and 0.34 for GPRP method, FP method and Yiu's method respectively. All the results show that our proposed method can both retain the accuracy and save the computation time in the localization system.

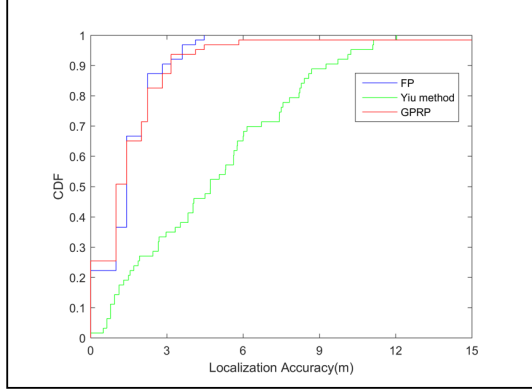


Fig.6. CDF of simulation localization error for different algorithms comparison

Accordingly, we employ our proposed method in the real environment. The area is divided into 6×9 blocks. Every block is around 1.4 m long and 1.2 m wide. All the experiments are tested on the 30% data for training and 70% for training. As usual, our proposed GPRP method is based on the derivation of blocks with equal width and height. In Fig.7, the RMSE for three methods are 2.28 m, 2.34 m and 3.09 for our proposed method, FP method and Yiu's method. The result reveals the same to the simulated environment, our proposed method can have the both advantages; time saving and accuracy training.

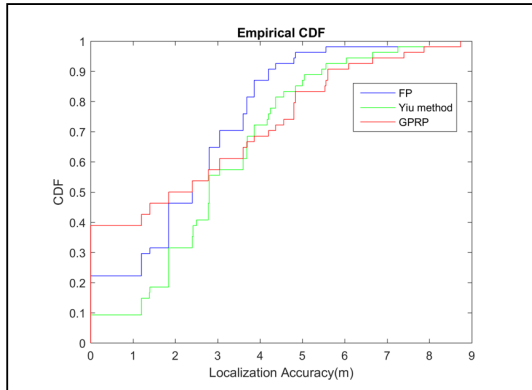


Fig.7. CDF of physical localization error for different algorithms comparison

V. CONCLUSION

Generally, the main challenge in RSS-based location positioning is the high sensitivity of this technique to environmental changes. Variations in the RSS measurement hinder the estimation accuracy. In other words, if the radio propagation signal strength was correlated with the distance between the transmitter and receiver, location determination would be a trivial problem. However, the relationship between these two parameters is dynamic rather than straightforward.

Hence, there are many methods to solve such kind of location prediction, when the position of receiver is changing. Fingerprinting database is basic method, which is the straight computation on the RSS signal collection. Time consumption and look-up table finding are the both bottlenecks.

In our study, we propose a new method, named GPRP method, for adapting the flexibility of Gaussian Processing, and referring the location with the simplified computation of Naive Bayes method. The computation time can be saved and efficiencies can be saved. In our experiments, we test our method on the simulated and real environment in the 55th building of Tianjin building. The experiments are beholding on the distinct data size, different kernels and accuracies testing. All the results show that our proposed method not only can retain the accuracy, but also can save the computation time in the location prediction.

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