

Glossary of Z notation

Names

a, b	identifiers
d, e	declarations (e.g., $a : A; b, \dots : B \dots$)
f, g	functions
m, n	numbers
p, q	predicates
s, t	sequences
x, y	expressions
A, B	sets
C, D	bags
Q, R	relations
S, T	schemas
X	schema text (e.g., $d, d p$ or S)

Definitions

$a == x$	Abbreviation definition
$a ::= b \dots$	Free type definition (or $a ::= b \langle\langle x \rangle\rangle \dots$)
$[a]$	Introduction of a given set (or $[a, \dots]$)
$a_$	Prefix operator
$_a$	Postfix operator
$_a_$	Infix operator

Logic

$true$	Logical true constant
$false$	Logical false constant
$\neg p$	Logical negation
$p \wedge q$	Logical conjunction
$p \vee q$	Logical disjunction
$p \Rightarrow q$	Logical implication ($\neg p \vee q$)
$p \Leftrightarrow q$	Logical equivalence ($p \Rightarrow q \wedge q \Rightarrow p$)
$\forall X \bullet q$	Universal quantification
$\exists X \bullet q$	Existential quantification
$\exists_1 X \bullet q$	Unique existential quantification
$let\ a ==\ x; \dots \bullet p$	Local definition

Sets and expressions

$x = y$	Equality of expressions
$x \neq y$	Inequality ($\neg (x = y)$)
$x \in A$	Set membership
$x \notin A$	Non-membership ($\neg (x \in A)$)
$\emptyset$	Empty set
$A \subseteq B$	Set inclusion
$A \subset B$	Strict set inclusion ($A \subseteq B \wedge A \neq B$)
$\{x, y, \dots\}$	Set of elements
$\{X \bullet x\}$	Set comprehension
$\lambda X \bullet x$	Lambda-expression – function
$\mu X \bullet x$	Mu-expression – unique value

$let\ a ==\ x; \dots \bullet y$ Local definition

$if\ p\ then\ x\ else\ y$ Conditional expression

$(x, y, \dots)$ Ordered tuple

$A \times B \times \dots$ Cartesian product

$P\ A$ Power set (set of subsets)

$P_1\ A$ Non-empty power set

$F\ A$ Set of finite subsets

$F_1\ A$ Non-empty set of finite subsets

$A \cap B$ Set intersection

$A \cup B$ Set union

$A \setminus B$ Set difference

$\bigcup A$ Generalized union of a set of sets

$\bigcap A$ Generalized intersection of a set of sets

$first\ x$ First element of an ordered pair

$second\ x$ Second element of an ordered pair

$\#A$ Size of a finite set

Relations

$A \leftrightarrow B$	Relation ($P(A \times B)$)
$a \mapsto b$	Maplet ((a, b))
$dom\ R$	Domain of a relation
$ran\ R$	Range of a relation
$id\ A$	Identity relation
$Q \circ R$	Forward relational composition
$Q \circ R$	Backward relational composition ($R \circ Q$)
$A \triangleleft R$	Domain restriction
$A \triangleleft R$	Domain anti-restriction
$R \triangleright A$	Range restriction
$R \triangleright A$	Range anti-restriction
$R(A)$	Relational image
$iter\ n\ R$	Relation composed n times
R^n	Same as $iter\ n\ R$
R^{-1}	Inverse of relation (R^{-1})
R^*	Reflexive-transitive closure
R^+	Irreflexive-transitive closure
$Q \oplus R$	Relational overriding ($(dom\ R \triangleleft Q) \cup R$)
$a \underline{R} b$	Infix relation

Functions

$A \rightarrowtail B$	Partial functions
$A \rightarrow B$	Total functions
$A \rightarrowtail B$	Partial injections
$A \rightarrow B$	Total injections
$A \twoheadrightarrowtail B$	Partial surjections
$A \twoheadrightarrow B$	Total surjections
$A \xrightarrow{\sim} B$	Bijjective functions
$A \rightarrowtail B$	Finite partial functions
$A \rightarrowtail B$	Finite partial injections
$f\ x$	Function application (or $f(x)$)

Numbers

$\mathbb{Z}$	Set of integers
$\mathbb{N}$	Set of natural numbers $\{0, 1, 2, \dots\}$
$\mathbb{N}_1$	Set of non-zero natural numbers $(\mathbb{N} \setminus \{0\})$
$m + n$	Addition
$m - n$	Subtraction
$m * n$	Multiplication
$m \text{ div } n$	Division
$m \bmod n$	Modulo arithmetic
$m \leq n$	Less than or equal
$m < n$	Less than
$m \geq n$	Greater than or equal
$m > n$	Greater than
$\text{succ } n$	Successor function $\{0 \mapsto 1, 1 \mapsto 2, \dots\}$
$m \dots n$	Number range
$\min A$	Minimum of a set of numbers
$\max A$	Maximum of a set of numbers

Sequences

$\text{seq } A$	Set of finite sequences
$\text{seq}_1 A$	Set of non-empty finite sequences
$\text{iseq } A$	Set of finite injective sequences
$\langle \rangle$	Empty sequence
$\langle x, y, \dots \rangle$	Sequence $\{1 \mapsto x, 2 \mapsto y, \dots\}$
$s \hat{\ } t$	Sequence concatenation
$\wedge / s$	Distributed sequence concatenation
$\text{head } s$	First element of sequence ($s(1)$)
$\text{tail } s$	All but the head element of a sequence
$\text{last } s$	Last element of sequence ($s(\#s)$)
$\text{front } s$	All but the last element of a sequence
$\text{rev } s$	Reverse a sequence
$\text{squash } f$	Compact a function to a sequence
$A \upharpoonright s$	Sequence extraction ($\text{squash}(A \triangleleft s)$)
$s \upharpoonright A$	Sequence filtering ($\text{squash}(s \triangleright A)$)
$s \text{ prefix } t$	Sequence prefix relation ($s \hat{\ } v = t$)
$s \text{ suffix } t$	Sequence suffix relation ($u \hat{\ } s = t$)
$s \text{ in } t$	Sequence segment relation ($u \hat{\ } s \hat{\ } v = t$)
$\text{disjoint } A$	Disjointness of an indexed family of sets
$A \text{ partition } B$	Partition an indexed family of sets

Bags

$\text{bag } A$	Set of bags or multisets ($A \rightarrow \mathbb{N}_1$)
$\square$	Empty bag
$[x, y, \dots]$	Bag $\{x \mapsto 1, y \mapsto 1, \dots\}$
$\text{count } C \ x$	Multiplicity of an element in a bag
$C \# x$	Same as $\text{count } C \ x$
$n \otimes C$	Bag scaling of multiplicity
$x \in C$	Bag membership
$C \sqsubseteq D$	Sub-bag relation
$C \oplus D$	Bag union

$C \ominus D$	Bag difference
$\text{items } s$	Bag of elements in a sequence

Schema notation

Vertical schema.

$\begin{array}{ c } \hline S \\ \hline d \\ \hline p \\ \hline \end{array}$	New lines denote ';' and '^'. The schema name and predicate part are optional. The schema may subsequently be referenced by name in the document.
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Axiomatic definition.

$\begin{array}{ c } \hline d \\ \hline p \\ \hline \end{array}$	The definitions may be non-unique. The predicate part is optional. The definitions apply globally in the document.
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Generic definition.

$\begin{array}{ c } \hline [a, \dots] = \\ \hline d \\ \hline p \\ \hline \end{array}$	The generic parameters are optional. The definitions must be unique. The definitions apply globally in the document.
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$S \triangleq [X]$	Horizontal schema
$[T; \dots \dots]$	Schema inclusion
$z.a$	Component selection (given $z : S$)
θS	Tuple of components
$\neg S$	Schema negation
$\text{pre } S$	Schema precondition
$S \wedge T$	Schema conjunction
$S \vee T$	Schema disjunction
$S \Rightarrow T$	Schema implication
$S \Leftrightarrow T$	Schema equivalence
$S \setminus (a, \dots)$	Hiding of component(s)
$S \upharpoonright T$	Projection of components
$S \circ T$	Schema composition (S then T)
$S \gg T$	Schema piping (S outputs to T inputs)
$S[a/b, \dots]$	Schema component renaming (b becomes a , etc.)
$\forall X \bullet S$	Schema universal quantification
$\exists X \bullet S$	Schema existential quantification
$\exists_1 X \bullet S$	Schema unique existential quantification

Conventions

$a?$	Input to an operation
$a!$	Output from an operation
a	State component before an operation
a'	State component after an operation
S	State schema before an operation
S'	State schema after an operation
ΔS	Change of state (normally $S \wedge S'$)
ΞS	No change of state (normally $[S \wedge S' \theta S = \theta S']$)

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